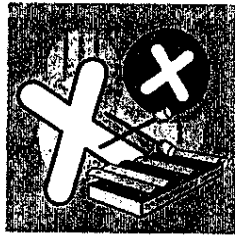


Chapter 5: Factoring



Notes
MS



5.1: Factoring by *GCF* and *Grouping*



5.2: Factoring Trinomials Form $x^2 + bx + c$



5.3: Factoring Trinomials Form $ax^2 + bx + c$



5.4: Factoring Difference of Two Squares
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5.5: Factoring Using Multiple Methods



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5.7: Simplify Rational Expressions



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5.1: Factoring by GCF and Grouping

What is factoring?

Find the prime factors of:

$$6$$

$$\wedge$$

$$2 \cdot 3$$

$$27$$

$$\wedge$$

$$3 \cdot 9$$

$$\wedge$$

$$3 \cdot 3 \cdot 3$$

$$250$$

$$\wedge$$

$$5 \cdot 50$$

$$\wedge$$

$$5 \cdot 2 \cdot 25$$

$$\wedge$$

$$5 \cdot 5 \cdot 5 \cdot 2$$

$$25 \cdot 10$$

$$\wedge \quad \wedge$$

$$5 \cdot 5 \cdot 5 \cdot 2$$

$$25x^2y^5$$

$$\wedge$$

$$5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$$

$$100ab^3$$

$$\wedge$$

$$10 \cdot 10$$

$$\wedge \quad \wedge$$

$$2 \cdot 5 \cdot 2 \cdot 5 \cdot a \cdot b \cdot b \cdot b$$

$$5 \cdot 5 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b \cdot b$$

$$45x^4y^2z$$

$$\wedge$$

$$5 \cdot 9$$

$$\wedge$$

$$5 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z$$

$$5 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z$$

The greatest common factor (GCF)- the largest common factor of the integers.

To Find the GCF:

1. Find the prime factorization of the terms
2. Find the common factors in each of the terms and the most of each in all terms
3. Multiply the most common factors in all of the terms.

Example: Find the GCF of: 36 and 90

$$\begin{array}{c}
 36 \\
 6 \cdot 6 \\
 2 \cdot 3 \cdot 2 \cdot 3 \\
 \\
 2 \cdot 2 \cdot 3 \cdot 3
 \end{array}$$

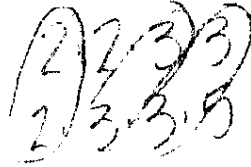
$$\begin{array}{c}
 90 \\
 9 \cdot 10 \\
 3 \cdot 3 \cdot 2 \cdot 5 \\
 \\
 2 \cdot 3 \cdot 3 \cdot 5
 \end{array}$$

The common factors are: 2, 3

The most 2's in common is: 2

The most 3's in common is: 3·3

The GCF is: $2 \cdot 3 \cdot 3 = 18$



Practice Example: Find the GCF of:

$$\begin{array}{c}
 24 \\
 \wedge \\
 2 \cdot 12 \\
 \wedge \wedge \\
 2 \cdot 2 \cdot 6 \\
 \wedge \wedge \wedge \\
 2 \cdot 2 \cdot 2 \cdot 3
 \end{array}$$

and 70

$$\begin{array}{c}
 70 \\
 \wedge \\
 7 \cdot 10 \\
 \wedge \wedge \\
 7 \cdot 2 \cdot 5 \\
 \\
 2 \cdot 5 \cdot 7 \\
 \\
 2 \cdot 5
 \end{array}$$

Common Factors: 2

2's in common: 1



Practice Example:

Find the GCF of:

$$\begin{array}{c}
 45 \\
 \wedge \\
 9 \cdot 5 \\
 \wedge \wedge \\
 3 \cdot 3 \cdot 5
 \end{array}$$

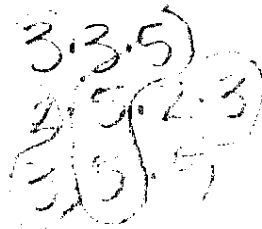
$$\begin{array}{c}
 60 \\
 \wedge \\
 12 \cdot 5 \\
 \wedge \wedge \\
 2 \cdot 5 \cdot 2 \cdot 3
 \end{array}$$

$$\begin{array}{c}
 75 \\
 \wedge \\
 3 \cdot 25 \\
 \wedge \wedge \\
 3 \cdot 5 \cdot 5
 \end{array}$$

Common Factors: 3, 5

3's in common: 1

5's in common: 1



Example: Find the GCF of: $9x^3y^2$ and $15xy^4$

$$9x^3y^2$$

$$15xy^4$$

$$3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$$

$$3 \cdot 5 \cdot x \cdot y \cdot y \cdot y \cdot y$$



The common factors are: 3, x, y

The most 3's in common is: 3

The most x's in common is: x

The most y's in common is: yy

The GCF is: $3xyy = 3xy^2$

Practice Example: Find the GCF of: $20x^5y^6$ and $150x^7y^3$

$$\begin{array}{c} \wedge \\ 5 \cdot 4 \\ 5 \cdot 2 \cdot 2 \end{array}$$

$$\begin{array}{c} \wedge \\ 10 \cdot 5 \\ 3 \cdot 5 \cdot 3 \end{array}$$

$$\boxed{10x^3y^3}$$

Practice Example:

Find the GCF of: $42a^3b^2$

$$\begin{array}{c} \wedge \\ 6 \cdot 7 \\ \wedge \quad | \\ 7 \cdot 3 \cdot 7 \end{array}$$

$63ab^5$

$$\begin{array}{c} \wedge \\ 9 \cdot 7 \\ \wedge \quad | \\ 3 \cdot 3 \cdot 7 \end{array}$$

$21a^5b^4$

$$\begin{array}{c} \wedge \\ 7 \cdot 3 \end{array}$$

$$7 \cdot 3 \cdot 21$$

$$\boxed{21ab^2}$$

Simplify: $3x(2x+5)$

$$6x^2 + 15x$$

$\frac{6x^2}{3x} = 2x$ $\frac{15x}{3x} = 5$
greatest # pulled out
3x (now divide)

Now work backwards to get the original back

$$3x(2x+5)$$

$$2x+5$$

Factor using Greatest Common Factor

Example: Factor- $25x^3y^3 + 15x^5y^6 - 5x^2y^7$



If we break down the polynomial into prime factors we would have:

$$5 \cdot 5xxxxyy + 5 \cdot 3xxxxxyyy - 5xyyyyyyy$$

What are the most factors you can pull out of all of the terms?

$$5x^2y^3$$

Then what remains once that is pulled out?

Divide by what you are factoring out to see what remains.

$$\frac{25x^3y^3}{5x^2y^3} + \frac{15x^5y^6}{5x^2y^3} - \frac{5x^2y^7}{5x^2y^3} =$$

$$5x + 3x^3y^3 - y^4$$

The factored form is: $5x^2y^3(5x - 3x^3y^3 - y^4)$

You can check by simplifying.

$$5x^2y^3(5x - 3x^3y^3 - y^4)$$

$$25x^3y^3 - 15x^5y^6 - 5x^2y^7$$

Practice Examples:

Factor:

$$\frac{10a^2b^4 + 15a^3b^2}{5a^2b^2 \quad 5a^2b^2}$$

$$2b^2 + 3a$$

$$5a^2b^2(2b^2 + 3a)$$

Factor:

$$\frac{3x^5y^{10} - 9x^7y^4 + 21x^2y^{12}}{3x^2y^6 \quad 3x^5y^4 \quad 3x^2y^8}$$

$$x^3y^6 - 3x^5 + 7y^8$$

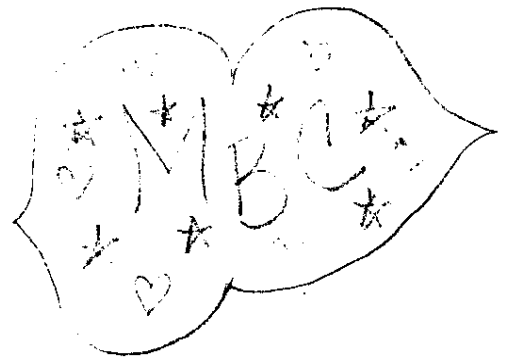
$$3x^2y^4(x^3y^6 - 3x^5 + 7y^8)$$

Factor:

$$\frac{65y^9v^{18} + 20y^{30}v^{20} + 30y^{18}v^4}{5y^9v^4 \quad 2y^{21}v^{16} \quad 5y^9v^4}$$

$$13v^{14} + 4y^{21}v^{16} + 6y^9$$

$$5y^9v^4(13v^{14} + 4y^{21}v^{16} + 6y^9)$$



Factor using Grouping

Factor:

$$3(x+2) - x(x+2)$$

$$\boxed{3(x+2) - x(x+2)} \quad C(3-x)$$

$$(x+2)(3-x)$$

Example: Factor- $2ys - 2yf + bs - bf$

Method 1 will not work because you can not pull out of all of the terms.

Since there are 4 terms, we can group 2 in common and another 2 in common.

$$2ys - 2yf \quad \text{and} \quad bs - bf$$

$$\begin{array}{cc} 2ys - 2yf & bs - bf \\ \downarrow & \downarrow \\ 2y(s-f) & b(s-f) \\ \downarrow & \downarrow \\ (s-f) & (2y+b) \end{array}$$

Now, we can pull out from each group:

$$2y(s-f) \quad \text{and} \quad b(s-f)$$

If this is factorable, then what is the parenthesis will match.

Pull out what they both have in common and see what remains.

$(s-f)(2y+b)$, this is the factored form.

You can check by simplifying.

Practice Example:

Factor:

$$xy + 3y - 6x - 18$$

$$xy + 3y \quad -6x - 18$$

$$y(x+3) \quad -6(x+3)$$

$$\boxed{(x+3)(y-6)}$$

To check. FOIL

Practice Examples:

Factor:

$$10k + 10m - km - m^2$$

$$\begin{aligned} &10k + 10m && -km - m^2 \\ &10(k+m) && -m(k+m) \\ &&& (k+m)(10-m) \end{aligned}$$

Factor:

$$x^2 - ax - x + a$$

$$\begin{aligned} &x^2 - ax && -x + a \\ &x(x-a) && -1(x-a) \\ &&& (x-a)(x-1) \end{aligned}$$

Factor:

$$\frac{5bx}{5} + \frac{5bz}{5} + \frac{25x}{5} + \frac{25z}{5}$$

$$\begin{aligned} &5(bx + bz + 5x + 5z) \\ &bx + bz && 5x + 5z \\ &b(x+z) && 5(x+z) \\ &\boxed{5(x+z)(b+5)} \end{aligned}$$

$$\begin{aligned} &5bx + 5bz && 25x + 25z \\ &5b(x+z) && 25(x+z) \\ &&& (x+z)(5b+25) \end{aligned}$$

$$\boxed{5(b+5)(x+z)}$$

What is factoring?

Break it down into Multiplication

- #1 GCF
- #2 GROUPING
- #3 TRINOMIAL EASY
- #4 TRINOMIAL HARD
- #5 Special Cases

5.1: Factoring by GCF and Grouping Practice Problems

1. Find the GCF of:

54, 72, 90

$$\begin{array}{ccc} \wedge & \wedge & \wedge \\ 10 \cdot 9 & 12 \cdot 6 & 10 \cdot 9 \\ \wedge \wedge & \wedge \wedge & \wedge \wedge \\ 2 \cdot 3 \cdot 3 & 2 \cdot 2 \cdot 3 & 2 \cdot 3 \cdot 3 \\ & 2 \cdot 2 \cdot 2 \cdot 3 & \\ & 2 \cdot 3 \cdot 3 & \end{array}$$

$18 = \text{GCF}$

2. Find the GCF of:

x^{10}, x^{15}

$$\boxed{x^{10}}$$

3. Find the GCF of:

$30x^6, 45x^9, 60x^{12}$

$$\begin{array}{ccc} \wedge & \wedge & \wedge \\ 3 \cdot 10 & 9 \cdot 5 & 10 \cdot 6 \\ \wedge \wedge & \wedge \wedge & \wedge \wedge \\ 3 \cdot 2 \cdot 3 & 3 \cdot 3 & 2 \cdot 3 \\ & 3 & \end{array}$$

5.3 $\boxed{15x^3}$

4. Find the GCF of:

$36a^5b, 60a^7b^4, 120a^9b^6$

$$\begin{array}{ccc} \wedge \wedge & \wedge \wedge & \wedge \wedge \\ 6 \cdot 6 & 3 \cdot 20 & 3 \cdot 40 \\ \wedge \wedge & \wedge \wedge & \wedge \wedge \\ 2 \cdot 3 & 2 \cdot 2 \cdot 5 & 2 \cdot 2 \cdot 2 \cdot 5 \\ & 2 & 2 \end{array}$$

2.2.3 $\boxed{12a^3b}$

5. Factor:

$$\frac{12x^5 - 24x^3}{12x^3 12x^3}$$

$$\boxed{12x^3(x^2 - 2)}$$

6. Factor:

$$\frac{24a^4b^6 + 40a^{10}b^5}{4a^4b^5 4a^4b^5}$$

$$\boxed{8a^4b^5(3a + 5a^6)}$$

7. Factor:

$$\frac{18a^3b^7 + 36a^4b^2 - 9ab}{9ab 9ab 9ab}$$

$$\boxed{9ab(2a^2b^6 + 4a^3b - 1)}$$

8. Factor:

$$\frac{40x^{10}y^{15}z^7 - 20x^7y^{30} - 60x^{22}y^{10}}{20x^7y^{10} 20x^7y^{10} 20x^7y^{10}}$$

$$\boxed{20x^7y^{10}(2x^3y^5z^7 - y^{20} - 3x^{15})}$$

5.1: Factoring by GCF and Grouping Practice Problems Continue

9. Factor: $x(x+3)+4(x+3)$

$$(x+3)(x+4)$$

10. Factor: $x^2(2x-7)-3x(2x-7)+2(2x-7)$

$$(2x-7)(x^2-3x+2)$$

11. Factor: $ax+ay+bx+by$

$$ax+ay \quad bx+by$$

$$a(x+y) \quad b(x+y)$$

$$(x+y)(a+b)$$

12. Factor: $x^2+x-2x-2$

$$x^2+x \quad -2x-2$$

$$x(x+1) \quad -2(x+1)$$

$$(x+1)(x-2)$$

13. Factor: $xy-xz-ay+az$

$$xy-xz \quad -ay+az$$

$$x(y-z) \quad -a(y-z)$$

$$(y-z)(x-a)$$

14. Factor: $15x^2-10xy+6x-4y$

$$15x^2-10xy \quad 6x-4y$$

$$5x(3x-2y) \quad 2(3x-2y)$$

$$(3x-2y)(5x+2)$$

15. Factor: $2a^2-3ab-2a+3b$

$$2a^2-3ab \quad -2a+3b$$

$$a(2a-3b) \quad -1(2a-3b)$$

$$(2a-3b)(a-1)$$

16. Factor: $\frac{20x^2}{2} + \frac{10xy}{2} - \frac{16x}{2} - \frac{8y}{2}$

$$2(10x^2+5xy-8x-4y)$$

$$10x^2+5xy \quad -8x-4y$$

$$5x(2x+y) \quad -4(2x+y)$$

$$2(2x+y)(5x-4)$$



5.2: Factoring Trinomials Form $x^2 + bx + c$

How do you factor trinomials in the form $x^2 + bx + c$?

Simplify: $(x+3)(x+4)$

Solution:

$$\begin{array}{r}
 (x+3)(x+4) \\
 \begin{array}{cccc}
 F & O & I & L \\
 x \cdot x & +4x & +3x & +3 \cdot 4 \\
 x^2 & +4x & +3x & +12
 \end{array} \\
 \begin{array}{ccc}
 F & O+I & L \\
 x^2 & +7x & +12
 \end{array}
 \end{array}$$

*multiples to the back
add to the middle*

Notice to get the first term, it is the **multiplication** of the **F** part of FOIL

Notice to get the inside term, it is the **addition** of the **O + I** of FOIL

Notice to get the outside term, it is the **multiplication** of **L** of FOIL.

Simplify: $(x-2)(x+5)$ $x^2 + 5x - 2x - 10$
 $x^2 + 3x - 10$

Example: Factor- $x^2 + 7x + 12$

Pulling out the GCF or Grouping will not work because you can not pull out of all of the terms and there is not an even number of terms to group.

Our goal is to factor $x^2 + 7x + 12$ into two factors that are being multiplied: () ()

To get the x^2 , the factors must be $x \cdot x$ in the front (F of FOIL).

To factor this polynomial where the leading coefficient is 1:

We need two numbers that will multiply to the last term of: + 12 (L of FOIL)

and add to the middle term of: + 7 (O + I of FOIL)

We look at the factors of +12 and find:

| | | |
|-----------|----------|---------|
| 1 · 12, | 2 · 6, | 3 · 4 |
| -1 · -12, | -2 · -6, | -3 · -4 |

Which pair will multiply to +12 and add to +7?

The numbers are: +3 and +4. $(3)(4) = 12$ $3 + 4 = 7$

******Make sure your signs are correct******

The factored form is: $(x + 3)(x + 4)$

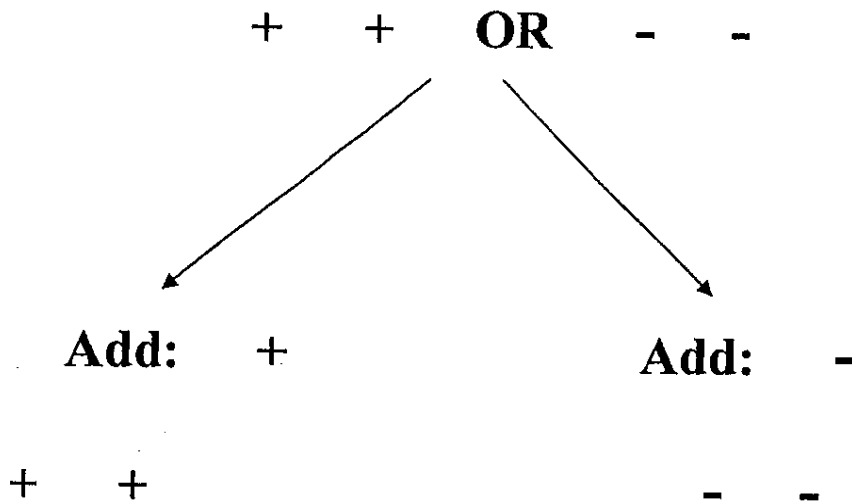
You can check by simplifying.

$$\begin{aligned} &(x + 3)(x + 4) \\ &x \cdot x + 4x + 3x + 3 \cdot 4 \\ &x^2 + 4x + 3x + 12 \\ &x^2 + 7x + 12 \end{aligned}$$

Know Your Signs

Multiply: +

Then the signs are:



Multiply: -

Then the signs are:

+ -

Practice Examples:

Factor: ^{add to} ₁₇ multiply to ⁺¹⁰

$$x^2 + 7x + 10$$

$$(x+2)(x+5)$$

Factor: ^{add -9} ₁₈ multiply to

$$x^2 - 9x + 18$$

$$(x-6)(x-3)$$

Factor: ^{add 29} ₅ multiply to

$$x^2 - 2x - 24$$

$$(x-6)(x+4)$$

~~10~~ 2
+-
6 · 4

Factor: ^{add 29} ₂₀ multiply to

$$x^2 + 29x - 30$$

$$(x-1)(x+30)$$

30 · 1
+-

Factor:
 $-x^2 + 14x - 45$

$$-(x^2 - 14x + 45)$$

$$-(x-9)(x-5)$$

Factor:
 $\frac{2x^2 - 12x + 16}{2}$

$$2(x^2 - 6x + 8)$$

$$2(x-4)(x-2)$$

How do you factor trinomials in the form $x^2 + bx + c$?

Use multiplying to the last # and
adding to the middle #

5.2: Factoring Trinomials Form $x^2 + bx + c$ Practice Examples

1. Factor: $x^2 + 4x + 3$

add to 4 multiply to 3

$$(x+3)(x+1)$$

3+1
++

$$\sqrt{x^2 + x + 3x + 3}$$

$$x^2 + 4x + 3$$

3. Factor: $x^2 - 6x + 9$

$$(x-3)(x-3)$$

3-3
--

$$\sqrt{x^2 - 6x + 9}$$

$$x^2 - 6x + 9$$

5. Factor: $x^2 - 5x - 24$

$$(x+3)(x-8)$$

8-3
+-

$$\sqrt{x^2 - 8x + 3x - 24}$$

$$x^2 - 5x - 24$$

7. Factor: $x^2 + 3x - 18$

$$(x+6)(x-3)$$

6-3
+-

$$\sqrt{x^2 - 3x + 6x - 18}$$

$$x^2 + 3x - 18$$

2. Factor: $x^2 + 7x + 10$

add to 7 multiply to 10

$$(x+5)(x+2)$$

5+2

$$\sqrt{x^2 + 2x + 5x + 10}$$

$$x^2 + 7x + 10$$

4. Factor: $x^2 - 8x + 12$

$$(x-6)(x-2)$$

6-2
--

$$\sqrt{x^2 - 8x + 12}$$

$$x^2 - 8x + 12$$

6. Factor: $x^2 - x - 20$

$$(x+4)(x-5)$$

5-4
+-

$$\sqrt{x^2 - 5x + 4x - 20}$$

$$x^2 - x - 20$$

8. Factor: $x^2 + 6x - 27$

$$(x+9)(x-3)$$

9-3
+-

$$\sqrt{x^2 - 3x + 6x - 27}$$

$$x^2 + 6x - 27$$

5.2: Factoring Trinomials Form $x^2 + bx + c$ Practice Examples Continue

9. Factor: $x^2 - 2x + 1$

$(x-1)(x-1)$

1 1
- -
+ +

$x^2 - 2x + 1$

11. Factor: $x^2 + 23x - 50$ 25, 2

$(x+25)(x-2)$

- +
- +

$x^2 + 23x - 50$

13. Factor: $x^2 + 27x + 72$ 24, 3

$(x+24)(x+3)$

+ +
+ +

$x^2 + 27x + 72$

15. Factor: $-x^2 - 7x + 8$

$-1(x^2 + 7x - 8)$ 8, 1

+ -
+ -

$-1(x+8)(x-1)$

$-1(x+8)(x-1)$

$-x^2 - 7x + 8$

10. Factor: $35 + x^2 - 12x$

$x^2 - 12x + 35$ 7, 5

$(x-7)(x-5)$

+ +
- -

$x^2 - 12x + 35$

12. Factor: $x^2 + 7x - 12$ 3, 4

$(x+3)(x-4)$

+ -
+ -

No Solution

Not Factorable

PRIME

14. Factor: $x^2 - 9x - 36$ 12, 3

$(x-12)(x+3)$

+ -
+ -

$x^2 - 9x - 36$

16. Factor: $3x^2 - 3x - 18$

$3(x^2 - x - 6)$ 3, 2

+ -
+ -

$3(x+2)(x-3)$

$3(x^2 - x - 6)$

$3(x+2)(x-3)$

$3x^2 - 3x - 18$

March 5th 2008



5.3: Factoring Trinomials Form $ax^2 + bx + c$

How do you factor trinomials in the form $ax^2 + bx + c$?

Example: Factor- $12x^2 - 23x + 5$

Method 1 or Method 2 or Method 3 will not work because:

You can not pull out of all of the terms (Method 1 Pull out GCF),

There is not an even number of terms to group (Method 2 Grouping), and

Since the leading coefficient is not 1, (Method 3 Trinomial $a = 1$)

You still need two numbers that multiply to the last term, but they will NOT add to the middle term coefficient because the leading coefficient is not 1.

There are two ways to factor this polynomial: Trial and Error and Change to grouping

First Method: Trial and Error

We know that when we factor $12x^2 - 23x + 5$, the factored form will be: () ()

Step 1: We will first list the possible factors:

We must get the leading term of $12x^2$

The possibilities are: $(1x)(12x)$ $(2x)(6x)$ $(3x)(4x)$

Step 2: We know that it must multiply to the last term of + 5:

The possibilities with signs are: $(+1)(+5)$ $(-1)(-5)$ ****Signs are VERY important****

Since the middle term is negative, and it must add to the middle term, then it must be: $(-1)(-5)$

Step 3: Then try different combinations and simply each one to try to get: $12x^2 - 23x + 5$

Ex. $(2x - 5)(6x - 1)$ $(x - 5)(12x - 1)$ $(3x - 1)(4x - 5)$ $(3x - 5)(4x - 1)$

$(3x - 5)(4x - 1)$, this is the factored form. You can check by simplifying.

Second Method: Change to grouping

Example: Factor- $12x^2 - 23x + 5$

In this method we want the trinomial to become a polynomial with four terms so we can perform grouping. There is a specific way to change the trinomial into a polynomial with four terms.

Step 1: Find the middle factors

Multiply the leading coefficient by the last constant term.

$12 \cdot 5 = +60$, we want to find factors that multiply to get this number (including the sign)

The factors must also add to the middle term coefficient, in this case -23 .

The factors that multiply to $+60$ and add to -23 are: -20 and -3

Step 2: Rewrite the polynomial

We found the two coefficients to replace the middle term to form a polynomial with 4 terms.

The trinomial $12x^2 - 23x + 5$ will now become:

$$12x^2 - 3x - 20x + 5$$

It does not matter which number goes first, but since we are going to group, you want to place factors next to terms they have something in common with.

For example, place the $-3x$ next to the $12x^2$ because they have a $3x$ in common and place the $-20x$ next to the 5 because they have a 5 in common.

Step 3: Use Grouping

$$\begin{array}{r} 12x^2 - 3x - 20x + 5 \\ 12x^2 - 3x \quad - 20x + 5 \\ 3x(4x - 1) \quad - 5(4x - 1) \\ (4x - 1)(3x - 5) \end{array}$$

$(4x - 1)(3x - 5)$, this is the factored form. You can check by simplifying.

Practice Example:

Factor: $\begin{array}{r} 6 \\ 2x^2 + 5x + 3 \end{array}$

$$\underline{2x^2} \quad \underline{+ 2x} \quad \underline{+ 3x} \quad \underline{3}$$

$$2x^2 + 2x \quad 3x + 3$$

$$2x(x+1) \quad 3(x+1)$$

$$(x+1)(2x+3)$$

$$\sqrt{3x^2 - 2x + 2x + 3} \quad 2x^2 + 5x + 3$$

Factor: $\begin{array}{r} -20 \\ 10x^2 + 13x - 3 \end{array}$

$$\underline{10x^2} \quad \underline{+ 15x} \quad \underline{- 2x} \quad \underline{- 3}$$

$$10x^2 + 15x \quad - 2x - 3$$

$$5x(2x+3x) \quad -1(2x+3)$$

$$(2x+3)(5x-1)$$

$$\sqrt{10x^2 - 2x + 15x - 3}$$

$$10x^2 + 13x - 3$$

Factor: $\begin{array}{r} 6 \cdot 3 \\ 9x^2 - 9x + 2 \end{array}$

$$\underline{9x^2} \quad \underline{- 3x} \quad \underline{+ 6x} \quad \underline{+ 2}$$

$$9x^2 - 3x \quad + 6x + 2$$

$$3x(3x-1) \quad +2(3x+1)$$

$$(3x+1)(3x-2)$$

$$\sqrt{9x^2 - 3x + 6x - 2}$$

Factor: $\begin{array}{r} -2 \\ 3x^2 - 7x - 6 \end{array}$

$$\underline{3x^2} \quad \underline{- 7x} \quad \underline{+ 2x} \quad \underline{- 6}$$

$$3x^2 - 9x \quad 2x - 6$$

$$3x(x-3) \quad 2(x-3)$$

$$(x-3)(3x+2)$$

$$\sqrt{3x^2 + 2x - 9x - 6}$$

$$3x^2 - 7x - 6$$

Factor:

$$-8x^3 + 2x^2 + 3x$$

$$x(-8x^2 + 2x + 3)$$

-24 6·4
+2 + -

take out the Negative

$$\underline{-8x^2} \quad \underline{-4x} \quad \underline{+6x} \quad \underline{3}$$

$$-8x^2 - 4x \quad 6x + 3$$

$$-4x(2x+1) \quad 3(2x+1)$$

$$x(-4x+3)(2x+1)$$

$$\checkmark (-4x^2 + 3x)(2x+1)$$

$$-8x^3 - 4x^2 + 6x^2 + 3$$

$$-8x^3 + 2x^2 + 3$$

Factor:

$$12x^5 - 17x^4 + 6x^3$$

$$x^3(12x^2 - 17x + 6)$$

+72 2·9
+17 -

$$\underline{12x^2} \quad \underline{-8x} \quad \underline{-9x} \quad \underline{6}$$

$$12x^2 - 8x \quad -9x + 6$$

$$4x(3x-2) \quad -3(3x-2)$$

$$x^3(3x-2)(4x-3)$$

$$\checkmark (3x^4 - 2x^3)(4x-3)$$

$$12x^5 - 9x^4 - 8x^4 + 6x^3$$

$$12x^5 - 17x^4 + 6x^3$$

$$\begin{array}{r} 12x^5 - 17x^4 + 6x^3 \\ \underline{4x(3x-2)} \\ 12x^5 - 8x^4 + 6x^3 \\ \underline{-9x(3x-2)} \\ 12x^5 - 17x^4 + 6x^3 \end{array}$$

How do you factor trinomials in the form $ax^2 + bx + c$?

Break it down into # 1-5 steps.

5.3: Factoring Trinomials Form $ax^2 + bx + c$ Practice Examples

1. Factor: $\overbrace{2x^2 + 7x + 3}^{+6} + 7$ ~~2.4~~
 $u \cdot 1$

$\underline{2x^2} \quad \underline{x} \quad \underline{6x} \quad \underline{3}$

$2x^2 + x \quad 6x + 3$

$x(2x+1) \quad 3(2x+1)$

$(x+3)(2x+1)$

$\sqrt{2x^2 + x + 6x + 3} \quad 2x^2 + 7x + 3$

3. Factor: $\overbrace{8x^2 + 10x - 3}^{-24} + 10$ ~~2.4~~
 $12 \cdot 2$

$\underline{8x^2} \quad \underline{-2x} \quad \underline{12x} \quad \underline{-3}$

$8x^2 - 2x \quad 12x - 3$

$2x(4x-1) \quad 3(4x-1)$

$(2x+3)(4x-1)$

$\sqrt{8x^2 - 2x + 12x - 3} \quad 8x^2 + 10x - 3$

5. Factor: $\overbrace{4x^2 + 4x - 15}^{-60} + 4$ ~~2.4~~
 $10 \cdot 10$
 $10 \cdot 10$

$\underline{4x^2} \quad \underline{10x} \quad \underline{-6x} \quad \underline{-15}$

$4x^2 + 10x \quad -6x - 15$

$2x(2x+5) \quad -3(2x+5)$

$(2x-3)(2x+5)$

$\sqrt{4x^2 + 10x - 6x - 15}$

$4x^2 + 4x - 15$

2. Factor: $\overbrace{6x^2 + 7x + 2}^{+12} + 7$ $3 \cdot 4$

$\underline{6x^2} \quad \underline{3x} \quad \underline{4x} \quad \underline{2}$

$6x^2 + 3x \quad 4x + 2$

$3x(2x+1) \quad 2(2x+1)$

$(3x+2)(2x+1)$

$\sqrt{6x^2 + 3x + 4x + 2} \quad 6x^2 + 7x + 2$

4. Factor: $\overbrace{6x^2 - x - 12}^{-72} + 10$ $8 \cdot 9$

$\underline{6x^2} \quad \underline{-9x} \quad \underline{8x} \quad \underline{-12}$

$6x^2 - 9x \quad 8x - 12$

$3x(2x-3) \quad 4(2x-3)$

$(3x+4)(2x-3)$

$\sqrt{6x^2 - 9x + 8x - 12} \quad 6x^2 - x - 12$

6. Factor: $\overbrace{5x^2 - 17x + 6}^{+30} + 17$ $15 \cdot 2$

$\underline{5x^2} \quad \underline{-15x} \quad \underline{-2x} \quad \underline{6}$

$5x^2 - 15x \quad -2x + 6$

$5x(x-3) \quad -2(x-3)$

$(5x-2)(x-3)$

$\sqrt{5x^2 - 15x - 2x + 6}$

$5x^2 - 17x + 6$

5.3: Factoring Trinomials Form $ax^2 + bx + c$ Practice Examples Continue

7. Factor: $25x^2 + 21x - 4$ $\begin{matrix} -100 \\ + 21 \end{matrix}$ $\begin{matrix} 25 \cdot 4 \\ + - \end{matrix}$

$$\begin{aligned} & \underline{25x^2} \quad \underline{25x} \quad \underline{-4x} \quad \underline{-4} \\ & 25x^2 + 25x \quad -4x - 4 \\ & 25x(x+1) \quad -4(x+1) \\ & \boxed{(25x-4)(x+1)} \end{aligned}$$

✓ $25x^2 + 25x - 4x - 4$ $25x^2 + 21x - 4$

8. Factor: $12x^2 - 25x + 12$ $\begin{matrix} +144 \\ -25 \end{matrix}$ $\begin{matrix} 12 \cdot 12 \\ + - \end{matrix}$ 9.16

$$\begin{aligned} & \underline{12x^2} \quad \underline{-9x} \quad \underline{-16x} \quad \underline{12} \\ & 12x^2 - 9x \quad -16x + 12 \\ & 3x(4x-3) \quad -4(4x-3) \\ & \boxed{(3x-4)(4x-3)} \end{aligned}$$

✓ $12x^2 - 9x - 16x + 12$ $12x^2 - 25x + 12$

9. Factor: $20x^2 - 31x - 7$ $\begin{matrix} -140 \\ -31 \end{matrix}$ $\begin{matrix} 55 \cdot 4 \\ + - \end{matrix}$

$$\begin{aligned} & \underline{20x^2} \quad \underline{4x} \quad \underline{-25x} \quad \underline{-7} \\ & 20x^2 + 4x \quad -25x - 7 \\ & 4x(5x+1) \quad -7(5x+1) \\ & \boxed{(4x-7)(5x+1)} \end{aligned}$$

✓ $20x^2 + 4x - 25x - 7$ $20x^2 - 31x - 7$

140
14 10
2 7 2 5
4 10
20 7
28 5
3 70
35 4

10. Factor: $2x^2 + 6x + 9$ $\begin{matrix} +18 \\ +3 \end{matrix}$ $\begin{matrix} 6 \cdot 3 \\ + + \end{matrix}$

$$\begin{aligned} & \underline{2x^2} \quad \underline{+6x} \quad \underline{+3x} \quad \underline{9} \\ & 2x^2 + 6x \quad 3x + 9 \\ & 2x(x+3) \quad 3(x+3) \\ & \boxed{(2x+3)(x+3)} \end{aligned}$$

12. Factor: $-3x^2 + 9x + 54$ $\begin{matrix} -18 \\ -3 \end{matrix}$ $\begin{matrix} 6 \cdot 3 \\ + - \end{matrix}$

$$\begin{aligned} & -3(x^2 - 3x - 18) \\ & \underline{x^2} \quad \underline{+3x} \quad \underline{-6x} \quad \underline{-18} \\ & x^2 + 3x \quad -6x - 18 \\ & x(x+3) \quad -6(x+3) \\ & \boxed{-3(x-6)(x+3)} \end{aligned}$$

11. Factor: $2x^3 + 4x^2 - 30x$ $\begin{matrix} -60 \\ +4 \end{matrix}$ $\begin{matrix} 10 \cdot 6 \\ + - \end{matrix}$

$$\begin{aligned} & x(2x^2 + 4x - 30) \\ & \underline{2x^2} \quad \underline{-6x} \quad \underline{10x} \quad \underline{-30} \\ & 2x^2 - 6x \quad 10x - 30 \\ & 2x(x-3) \quad 10(x-3) \\ & \boxed{x(2x+10)(x-3)} \end{aligned}$$

pull out 2x
out of 2x

✓ $(2x^2 + 10x)(x-3)$ $2x^3 - 6x^2 + 10x^2 - 30x$

✓ $(-3x+18)(x+3)$ $-3x^2 - 9x + 18x + 54$

March 10th 2008



5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials

How do you factor difference of two squares?

Binomial (+/-) Special Case (Difference of Two Squares)

Example: Factor- $4x^2 - 9$

Method 1 or Method 2 or Method 3 or Method 4 will not work because you can not pull out of all of the terms, there is not an even number of terms to group, and it is not a trinomial.

This is a Special Case because:

- 1) There are only 2 terms
- 2) There is a (-) in the middle
- 3) You can take the square root of both terms



- #1 Pull out GCF
- #2 GROUPING
- #3 TRINOMIAL EASY
- #4 TRINOMIAL HARD
- #5 SPECIAL CASE

If all the criteria above represented, then you have **difference of squares**.

To factor this polynomial, you take the square root of both terms:

2x and 3

$$x^2 - 9$$

$$(2x+3)(2x-3)$$

Since there is no middle term, it must have canceled each other out.

$$4x^2 - 6x + 6x - 9$$

$$4x^2 - 9$$

Therefore, you make one factor with (+) and another factor with (-).

$(2x+3)(2x-3)$, this is the factored form.

You can check by simplifying.

$$(2x+3)(2x-3)$$

$$4x^2 - 6x + 6x - 9$$

$$4x^2 - 9$$

$$110x^2 - 49$$

$$(4x+7)(4x-7)$$

$$160x^2 - 28x + 28x - 49$$

$$160x^2 - 49$$

Practice Example:

Factor:

$100 - x^2$

#1 #3
#2

~~WAAAAA~~
 $(10-x)(10+x)$

$\checkmark 100 - 10x + 10x - x^2$
 $100 - x^2$

Factor:

$49x^2 + 36$

#1
#2

Doesn't work

Factor:

$16x^2 - 25$

#1 #3
#2

$(4x+5)(4x-5)$

$\checkmark 16x^2 - 20x + 20x - 25$
 $16x^2 - 25$

Factor:

$8x^2 - 50$

#1 #3
#2

$2(2x+5)(2x-5)$

$\checkmark (4x+10)(2x-5)$
 $8x^2 - 20x + 20x - 50$
 $8x^2 - 50$

Factor:

$36x^2 - 81y^2$

#1 #3
#2 - pull out a 9

$(6x+9y)(6x-9y)$

$\checkmark 36x^2 - 36xy + 36xy - 81y^2$
 $36x^2 - 81y^2$

Factor:

$a^2 - b^2$

#1 #3
#2

$(a+b)(a-b)$

$\checkmark a^2 - ab + ab - b^2$
 $a^2 - b^2$

$9(4x^2 - 9y^2)$
 $9(2x-3y)(2x+3y)$

$\checkmark (18x - 27y)$
 $(2x-3y)$
 $36x^2 + 54xy - 54xy - 81y^2$
 $-81y^2$

How do you factor difference of two squares?

Check for all 3 requirements then make a

Final answer (2 terms, subtraction, square root)

Factoring Perfect Square Trinomial

Example: Factor- $x^2 + 10x + 25$

$x^2 + 10x + 25$ is a perfect square trinomial because:

The first term: x^2 is the square of x

$$x \cdot x = x^2$$

The last term: 25 is the square of 5:

$$5^2 = 25$$

The second term: $10x$ is twice the product of x and 5:

$$2(5)(x) = 10x$$

A trinomial is a perfect square trinomial if:

$$\begin{array}{r} x^2 + 10x + 25 \\ \sqrt{x^2} + 10x + \sqrt{25} \\ x \qquad \qquad 5 \\ \quad + 2(5x) \\ x \quad + \quad 5 \end{array}$$

The factored form of: $x^2 + 10x + 25$ is: $(x+5)^2$
 $(x+5)(x+5) = (x+5)^2$

Example: Factor- $9x^2 - 42x + 49$

$9x^2 - 42x + 49$ is a perfect square trinomial because:

The first term: $9x^2$ is the square of $3x$

$$3x \cdot 3x = 9x^2$$

The last term: 49 is the square of 7:

$$7^2 = 49$$

The second term: $-42x$ is - twice the product of $3x$ and 7:

$$-2(3x)(7) = -42x$$

A trinomial is a perfect square trinomial if:

$$\begin{array}{r} 9x^2 - 42x + 49 \\ \sqrt{9x^2} - 42x + \sqrt{49} \\ 3x \qquad \qquad 7 \\ \quad - 2(3x)(7) \\ 3x \quad - \quad 7 \end{array}$$

$$(3x-7)^2$$

The factored form of: $9x^2 - 42x + 49$ is: $(3x-7)(3x-7) = (3x-7)^2$

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials Practice Problems

~~19~~
~~2~~
~~21~~

Factor the following:

1. $x^2 - 16$ #1 #2 #3

$(x+4)(x-4)$

~~$\sqrt{x^2 - 16} = x - 4$~~
 ~~$x = 16$~~

2. $25 - x^2$ #1 #2 #3

$(5+x)(5-x)$

~~$\sqrt{25 - 5x + x^2} = 5 - x$~~
 ~~$5 - x^2$~~

3. $9x^2 - 49$ #1 #2 #3

$(3x+7)(3x-7)$

~~$\sqrt{9x^2 - 21x + 49} = 3x - 7$~~
 ~~$3x = 49$~~

4. $x^2 + 4$ #1 #2

DIFFICULT WORK

5. $-x^2 + 100$

$-1(x^2 - 100)$ #1 #2 #3

$-1(x+10)(x-10)$

6. $2x^2 - 8$

$2(x^2 - 4)$ #1 #2 #3

7. $4x^2 - 36$

$4(x^2 - 9)$ #1 #2 #3

$4(x+3)(x-3)$

~~$4(x+12)(x-3)$~~

~~$4x^2 - 12x + 12x - 36$~~

~~$4x^2 - 36$~~

8. $c^2 - d^2$ #1 #2 #3

$(c+d)(c-d)$

9. $121x^2 - 81y^4$

$(11x+9y)(11x-9y)$

~~$\sqrt{121x^2 - 99xy + 99xy - 81y^4} = 11x - 9y^2$~~

5.4: Factoring Difference of Two Squares and Factor Perfect Square Trinomials Practice Problems Continue

Factor the following:

10. $x^2 + 18x + 81$

$x \quad \boxed{+9} \quad 9$

$(x+9)^2$

✓ $(x+9)(x+9)$
 $x^2 + 9x + 9x + 81$
 $x^2 + 18x + 81$

11. $x^2 + 20x + 100$

$x \quad \boxed{+10} \quad 10$

$(x+10)^2$

12. $4x^2 + 28x + 49$

$2x \quad \boxed{+7} \quad 7$

$(2x+7)^2$

✓ $(2x+7)(2x+7)$
 $4x^2 + 14x + 14x + 49$
 $4x^2 + 28x + 49$

13. $x^2 - 10x + 25$

$x \quad \boxed{-5} \quad 5$

$(x-5)^2$

✓ $(x-5)(x-5)$
 $x^2 - 5x - 5x + 25$
 $x^2 - 10x + 25$

14. $x^2 - 8x - 16$

$(x + \quad)(x - \quad)$ ^{8:2} _{4:4} ^{16:1}

Not Factorable

15. $9x^2 - 12x + 4$

$3x \quad \boxed{-2} \quad 2$

$(3x-2)^2$

✓ $(3x-2)(3x-2)$
 $9x^2 - 6x - 6x + 4$
 $9x^2 - 12x + 4$

16. $25x^2 + 60x + 36$

$5x \quad \boxed{+6} \quad 6$

$(5x+6)^2$

✓ $(5x+6)(5x+6)$
 $25x^2 + 30x + 30x + 36$
 $25x^2 + 60x + 36$

17. $49x^2 - 54x + 16$

$7x \quad \boxed{-4} \quad 4$

$(7x-4)^2$

✓ $(7x-4)(7x-4)$
 ~~$49x^2 - 28x - 28x + 16$
 $49x^2 - 54x + 16$~~
 Not Factorable

18. $9x^2 - 66xy + 121y^2$

$3x \quad \boxed{-11} \quad 11$

$(3x-11)^2$

✓ $(3x-11)(3x-11)$
 $9x^2 - 33xy + 33xy + 121y^2$
 $9x^2 - 66xy + 121y^2$



5.5: Factoring Using Multiple Methods

If sometimes you need to factor using multiple methods,
what process should you follow?

Steps for Factoring Polynomial

Check each method in this order:

1) Pull out the GCF

Ex. $25x^3y^3 + 15x^5y^6 - 5x^2y^7$
 $5x^2y^3(5x + 3x^3y^3 - y^4)$

$5x^2y^3(5x + 3x^3y^3 - y^4)$

2) Grouping (Hint: 4 Terms or terms that can be paired)

Ex. $2ys - 2yf + bs - bf$
 $(s-f)(2y+b)$

$2ys - 2yf$ $bs - bf$
 $2y(s-f)$ $(s-f)(b)$
 $(s-f)(2y+b)$

3) Trinomial where leading coefficient is 1

Ex. $x^2 + 7x + 12$
 $(x+3)(x+4)$

$(x-3)(x-4)$

multiply to +12 3 & 4
- Add to +7 (+) (+)

$(s-f)(2y+b)$

4) Trinomial where the leading coefficient is not 1

Ex. $12x^2 - 23x + 5$
 $(3x-5)(4x-1)$

multiplied to 60
add to -23

10 & 6
30 & 2
20 & 3

$12x^2 - 3x - 20x + 5$

$12x^2 - 3x - 20x + 5$

$3x(4x-1) - 5(4x-1)$

$(4x-1)(3x-5)$

5) Difference of Squares

Ex. $4x^2 - 9$
 $(2x+3)(2x-3)$

$4x^2 - 0x + 9$
 $(2x+3)(2x-3)$

If sometimes you need to factor using multiple methods,
what process should you follow?

Start with #1 & go through #6

Practice Examples:

Factor:

$$8x^2 - 98$$

$$2(4x^2 - 49) \begin{matrix} \#1 \\ \#2 \#3 \end{matrix}$$

$$\boxed{2(2x-7)(2x+7)}$$

$$\textcircled{1} (4x-14)(2x+7) \begin{matrix} 2 \\ 2 \\ 1 \\ 1 \end{matrix}$$

$$8x^2 + 28x - 14x - 98$$

$$8x^2 - 98$$

Factor:

$$-4x^5y^2 - 12x^4y^3 - 9x^3y^4$$

$$-x^3y^2(4x^2 + 12xy + 9y^2)$$

$$2x \boxed{+2} 3y$$

$$\boxed{-x^3y^2(2x+3y)^2}$$

Factor:

$$-4x^2 + 32x + 4x^3$$

$$4x^3 - 4x^2 + 32x$$

$$\boxed{4x(x^2 - x + 8)}$$

+8 multiply
-1 Add

$$(x)(x)$$

Final
ANSWER

doesn't work

Factor:

$$-24xy + 42xy^2 + 12y^3x$$

$$12xy^3 + 42xy^2 - 24xy$$

$$6xy(2y^2 + 7y - 4) \begin{matrix} -8 & 81 \\ +7 & + \end{matrix}$$

$$2y^2 - 1y + 8y - 4$$

$$2y^2 - 1y \quad 8y - 4$$

$$y(2y-1) \quad 4(2y-1)$$

$$\boxed{6xy(2y-1)(y+4)}$$

5.5: Factoring Using Multiple Methods Practice Problems

1. Factor:

$$7x^2 - 28$$

$$7(x^2 - 4) \text{ \#1 \#2 \#3}$$

$$\boxed{7(x+2)(x-2)}$$

$$\checkmark (7x + 14)(x - 2)$$

$$7x^2 - 14x + 14x - 28$$

$$7x^2 - 28$$

2. Factor:

$$2x^3 - 16x^2 + 24x$$

$$2x(x^2 - 8x + 12)$$

$$\boxed{2x(x-6)(x-2)}$$

$$\checkmark 2x(x-6)(x-2)$$

$$2x^2 - 16x(x-2)$$

$$2x^3 - 4x^2 - 12x^2 + 24x$$

$$2x^3 - 16x^2 + 24x$$

3. Factor:

$$x^2(a+b) - 25(a+b)$$

$$\boxed{(x^2 - 25)(a+b)} \quad \boxed{(x+5)(x-5)}$$

$$\checkmark ax^2 + bx^2 - 25a - 25b$$

$$ax^2 + bx^2 - 25a - 25b$$

$$x^2(a+b) - 25(a+b)$$

4. Factor:

$$4x^3 + 12x^2 - 9x - 27$$

$$4x^3 + 12x^2 - 9x - 27$$

$$4x^2(x+3) - 9(x+3)$$

$$\boxed{(4x^2 - 9)(x+3)}$$

$$\checkmark 4x^3 + 12x^2 - 9x - 27$$

5. Factor:

$$\frac{-6x - 5x^2 + 6x^3}{6x^3 - 5x^2 - 6x}$$

$$x \left(\frac{6x^2 - 5x - 6}{6x^2 - 5x - 6} \right) = \frac{30}{5} = 6$$

$$\frac{6x^2 - 9x + 4x - 6}{6x^2 - 9x + 4x - 6}$$

$$6x^2 - 9x \quad 4x - 6$$

$$3x(2x-3) \quad 2(2x-3)$$

$$\boxed{x(3x+2)(2x-3)}$$

$$\checkmark (6x^2 - 9x)(2x - 3)$$

$$6x^3 - 9x^2 + 4x^2 - 6x$$

$$6x^3 - 5x^2 - 6x$$

6. Factor:

$$a^2bc^2 - 10a^2bc + 24a^2b$$

$$a^2b(c^2 - 10c + 24)$$

$$\boxed{a^2b(c-6)(c-4)}$$

$$\checkmark (a^2bc - 6a^2b)(c-4)$$

$$a^2bc^2 - 4a^2bc - 6a^2bc + 24a^2b$$

$$a^2bc^2 - 10a^2bc + 24a^2b$$



March 12th 2008

5.6: Solve Quadratic Equations by Factoring

How do you solve quadratic equations by factoring?

Example: Solve: $2x^2 + 5x - 3 = 0$ by Factoring

In this method, MAKE SURE that the quadratic equation is equal to 0

Step 1: Factor the quadratic equation

$$2x^2 + 5x - 3 = 0$$
$$(2x - 1)(x + 3) = 0$$

Step 2: Set each factor equal to 0

$$(2x - 1)(x + 3) = 0$$
$$2x - 1 = 0 \quad x + 3 = 0$$

Step 3: Solve each equation

$$2x - 1 = 0$$

| | | | |
|----------|----------|--|---------------|
| +1 | +1 | | $x + 3 = 0$ |
| $2x$ | $= 1$ | | $-3 \quad -3$ |
| $\div 2$ | $\div 2$ | | $x = -3$ |
| x | $= 0.5$ | | |

Step 4: Check your answers

Answers are $x = 0.5$ and $x = -3$

Substitute each answer individually into the original equation:

$$2x^2 + 5x - 3 = 0$$
$$2(0.5)^2 + 5(0.5) - 3 = 0$$
$$0 = 0$$

$$2x^2 + 5x - 3 = 0$$
$$2(-3)^2 + 5(-3) - 3 = 0$$
$$0 = 0$$

Example: Solve the quadratic equation: $x^2 - 4x - 12 = 0$

Step 1: Set the quadratic equation equal to zero

$$x^2 - 4x - 12 = 0$$

Step 2: Factor the quadratic equation

$$(x + 2)(x - 6) = 0$$

$$(x - 6)(x + 2) = 0$$

$$x + 2 = 0 \quad x - 6 = 0$$

Step 3: Set each factor equal to zero

$$-2 \quad -2 \quad +6 \quad +6$$

$$x - 6 = 0$$

$$x + 2 = 0$$

$$\boxed{x = -2}$$

$$\boxed{x = 6}$$

Step 4: Solve each factor

$$x - 6 = 0$$

$$x + 2 = 0$$

$$+6 \quad +6$$

$$-2 \quad -2$$

$$x = 6$$

$$x = -2$$

Step 5: Check the solutions

Algebraically

$$x^2 - 4x - 12 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(6)^2 - 4(6) - 12 = 0$$

$$(-2)^2 - 4(-2) - 12 = 0$$

$$36 - 24 - 12 = 0$$

$$4 + 8 - 12 = 0$$

$$0 = 0$$

$$0 = 0$$

Practice Example: Solve: $6x^2 - 7x + 2 = 0$ by Factoring

$$6x^2 - 7x + 2 = 0$$

$$6x^2 - 3x - 4x + 2$$

$$6x^2 - 3x - 4x + 2$$

$$3x(2x - 1) - 2(2x - 1)$$

$$(3x - 2)(2x - 1) = 0$$

$$3x - 2 = 0 \quad 2x - 1 = 0$$

$$+2 \quad +2$$

$$+1 \quad +1$$

$$\frac{2x}{3} = \frac{2}{3}$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$\boxed{x = \frac{2}{3}}$$

$$\boxed{x = \frac{1}{2}}$$

EXAMPLE: Solve the quadratic equation: $3x(x+1) = 2x+2$

Step 1: Set the quadratic equation equal to zero

$$\begin{aligned} 3x(x+1) &= 2x+2 \\ 3x^2+3x &= 2x+2 \\ -2x \quad -2x & \\ 3x^2+x &= 2 \\ \frac{}{2} \quad -\frac{2}{2} & \end{aligned}$$

$$\boxed{3x^2+x-2=0}$$

Step 2: Factor the quadratic equation

$$\begin{aligned} 3x^2+x-2 &= 0 \quad -\frac{2}{1} \cdot 3 \\ \frac{3x^2+3x-2x-2}{3x^2+3x-2x-2} & \\ \frac{3x(x+1)-2(x+1)}{3x(x+1)-2(x+1)} & \end{aligned} \quad (3x-2)(x+1)=0$$

Step 3: Set each factor equal to zero

$$3x-2=0 \quad x+1=0$$

Step 4: Solve each factor

$$\begin{aligned} 3x-2 &= 0 & x+1 &= 0 \\ +2 \quad +2 & & -1 \quad -1 & \\ \frac{3x}{3} &= \frac{2}{3} & \boxed{x} &= -1 \\ \boxed{x} &= \frac{2}{3} & & \end{aligned}$$

Step 5: Check the solutions

$$\begin{aligned} \checkmark (3(-1)-2)(-1+1) &= 0 \\ (-3-2)(-1+1) &= 0 \\ (-5)(0) &= 0 \\ 0 &= 0 \end{aligned}$$

How do you solve quadratic equations by factoring?

Set it equal to zero

5.6: Solve Quadratic Equations by Factoring Practice Problems

Solve:

1. $(x+2)(x-3)=0$
 $x+2=0$ $x-3=0$
 $-2-2$ $+3+3$
 $x=-2$ $x=3$

2. $x(x+6)(2x-1)=0$
 $x=0$ $x+6=0$ $2x-1=0$
 $-6-6$ $+1+1$
 $x=6$ $\frac{2x}{2}=\frac{1}{2}$
 $x=\frac{1}{2}$

$x=0$ $x=6$ $x=\frac{1}{2}$

3. $x^2-3x=0$

$x(x-3)=0$
 $x=0$ $x-3=0$
 $+3+3$
 $x=3$

$x=0$ $x=3$

4. $5x^2+10x=0$

$5x(x+2)=0$
 $5x=0$ $x+2=0$
 $\frac{5}{5}$ $-2-2$
 $x=0$ $x=-2$

$x=0$ $x=-2$

5. $4x(x+7)-3(x+7)=0$

$(x+7)=0$ $x+7=0$
 $+7+7$ $-7-7$
 $\frac{4x-3}{4}=\frac{3}{4}$ $x=-7$

$x=\frac{3}{4}$ $x=-7$

6. $8x^2+12x+10x+15=0$

$8x^2+12x$ $10x+15$
 $4x(2x+3)$ $5(2x+3)$
 $(4x+5)(2x+3)=0$
 $4x+5=0$ $2x+3=0$
 $-5-5$ $-3-3$
 $\frac{4x}{4}=\frac{-5}{4}$ $\frac{2x}{2}=\frac{-3}{2}$
 $x=-\frac{5}{4}$ $x=-\frac{3}{2}$

$x=-\frac{5}{4}$ $x=-\frac{3}{2}$

5.6: Solve Quadratic Equations by Factoring Practice Problems Continue

Solve:

7. $x^2 - 11x + 30 = 0$ $+30$
 ± 11 $6 \cdot 5$

$$(x-5)(x-6) = 0$$

$$x-5=0 \quad x-6=0$$

$$+5 \quad +6$$

$$x=5 \quad x=6$$

$$\boxed{x=5 \quad x=6}$$

8. $x^2 - 100 = 0$

$$(x+10)(x-10) = 0$$

$$x+10=0 \quad x-10=0$$

$$-10 \quad +10$$

$$x=-10 \quad x=10$$

$$\boxed{x=-10 \quad x=10}$$

9. $12x^2 - 17x - 5 = 0$ -60 $12 \cdot 5$ $30 \cdot 2$
 ± 17 $10 \cdot 4$ $20 \cdot 3$
 $-$ $+$

$$12x^2 + 3x - 20x - 5$$

$$12x^2 + 3x \quad -20x - 5$$

$$3x(4x+1) - 5(4x+1)$$

$$(3x-5)(4x+1) = 0$$

$$3x-5=0 \quad 4x+1=0$$

$$+5 \quad -1$$

$$\frac{3x}{3} = \frac{5}{3} \quad \frac{4x}{4} = \frac{-1}{4}$$

$$x = \frac{5}{3} \quad x = -\frac{1}{4}$$

$$\boxed{x = \frac{5}{3} \quad x = -\frac{1}{4}}$$

10. $x^2 - 4x - 30 = 0$ -30 $2 \cdot 4$
 ± 2 -30 $2 \cdot 4$
 -4 -4 $+8$ $+8$
 $-$ $+$

$$x^2 - 4x - 30 = 0$$

$$(x+4)(x-8) = 0$$

$$x+4=0 \quad x-8=0$$

$$-4 \quad +8$$

$$x=-4 \quad x=8$$

$$\boxed{x=-4 \quad x=8}$$

11. $-10 + 6x^2 = 11x$ -10 $4 \cdot 5$
 $-11x$ $-11x$
 $-$ $+$

$$6x^2 - 11x - 10 = 0$$

$$6x^2 + 4x - 15x - 10$$

$$6x^2 + 4x \quad -15x - 10$$

$$2x(3x+2) - 5(3x+2)$$

$$(2x-5)(3x+2) = 0$$

$$2x-5=0 \quad 3x+2=0$$

$$+5 \quad -2$$

$$\frac{2x}{2} = \frac{5}{2} \quad \frac{3x}{3} = \frac{-2}{3}$$

$$x = \frac{5}{2} \quad x = -\frac{2}{3}$$

$$\boxed{x = \frac{5}{2} \quad x = -\frac{2}{3}}$$

12. $14x^3 + 19x^2 + 5x = 8x$ $-8x$ $2 \cdot 2$
 $-8x$ -42 $2 \cdot 2$
 $+19$ $+$ $-$

$$14x^3 + 19x^2 - 3x = 0$$

$$x(14x^2 + 19x - 3) = 0$$

$$x(\frac{14x^2 - 2x + 21x - 3}{2}) = 0$$

$$14x^2 - 2x \quad 21x - 3$$

$$2x(7x-1) \quad 3(7x-1)$$

$$(2x+3)(7x-1)$$

$$x=0 \quad 2x+3=0 \quad 7x-1=0$$

$$-3 \quad +1$$

$$\frac{2x}{2} = \frac{-3}{2} \quad \frac{7x}{7} = \frac{1}{7}$$

$$x = -\frac{3}{2} \quad x = \frac{1}{7}$$

$$\boxed{x=0 \quad x = -\frac{3}{2} \quad x = \frac{1}{7}}$$



5.7: Simplify Rational Expressions

How do you simplify rational expressions?

Example: Simplify- $\frac{10}{35}$

Solution:

Step 1: Factor the numerator and denominator

$$\frac{10}{35} = \frac{2 \cdot 5}{7 \cdot 5}$$

Step 2: Simplify

$$\frac{2 \cdot \cancel{5}}{7 \cdot \cancel{5}}$$

Step 3: Final Answer

$$\frac{2}{7}$$

Practice Examples:

Simplify- $\frac{45}{63} \frac{\cancel{3} \cdot \cancel{3} \cdot 5}{\cancel{3} \cdot \cancel{3} \cdot 7} \boxed{\frac{5}{7}}$

Simplify- $\frac{60}{140} \frac{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 7} \boxed{\frac{3}{7}}$

Simplify- $\frac{25x^4y}{10x^2y^5} \frac{\cancel{5} \cdot 5x^4y}{2 \cdot \cancel{5}x^2y^5} \boxed{\frac{5x^2}{2y^4}}$

Simplify- $\frac{14a^7b^4}{20a^3b^5} \frac{\cancel{2} \cdot 7a^7b^4}{5 \cdot \cancel{2}a^3b^5} \boxed{\frac{7a^4}{10b}}$

Example: Simplify- $\frac{3x^2 - 8x - 3}{x^2 - 9}$

Solution:

Step 1: Factor the numerator and denominator

$$\frac{3x^2 - 8x - 3}{x^2 - 9} = \frac{(3x+1)(x-3)}{(x+3)(x-3)}$$

Step 2: Simplify

$$\frac{(3x+1)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$$

Step 3: Final Answer

$$\frac{3x+1}{x+3}$$

Practice Examples:

Simplify- $\frac{2x-14}{x^2-49}$

$$\frac{2\cancel{(x-7)}}{\cancel{(x+7)}\cancel{(x-7)}}$$

$$\boxed{\frac{2}{x+7}}$$

Simplify- $\frac{x^2+3x+2}{x^2+x-2}$

$$\frac{\cancel{(x+2)}\cancel{(x+1)}}{\cancel{(x+2)}(x-1)}$$

$$\boxed{\frac{x+1}{x-1}}$$

Practice Examples:

Simplify- $\frac{x^2 - 4}{x^2 - 2x - 8}$

$$\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-4)} \quad \boxed{\frac{x-2}{x-4}}$$

Simplify- $\frac{x^2 + 3x + 2}{x^2 + x - 2}$

$$\frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-1)} \quad \boxed{\frac{x+1}{x-1}}$$

Simplify- $\frac{6x-30}{5-x} = \frac{6x-30}{-x+5}$

$$\frac{6\cancel{(x-5)}}{-1\cancel{(x-5)}} \quad \boxed{-6}$$

Simplify- $\frac{6x^2 - 13x + 6}{3x^2 + x - 2}$

$$\begin{array}{l} 6x^2 - 13x + 6 \overset{30}{-13} \\ \underline{6x^2 - 4x} \quad \underline{-9x + 6} \\ 2x(3x-2) \quad -3(3x-2) \\ (2x-3)(3x-2) \end{array} \quad \begin{array}{l} 3x^2 + x - 2 \overset{-6}{+13} \cdot 2 \\ \underline{3x^2 + 3x} \quad \underline{-2x - 2} \\ 3x(x+1) \quad -2(x+1) \\ (3x-2)(x+1) \end{array}$$

$$\frac{(2x-3)\cancel{(3x-2)}}{\cancel{(3x-2)}(x+1)} \quad \boxed{\frac{2x-3}{x+1}}$$

How do you simplify rational expressions?

Factor and cancel out

5.7: Simplify Rational Expressions Practice Problems

Simplify:

$$1. \quad \frac{7}{77} \quad \frac{\cancel{7}}{\cancel{7} \cdot 11} \quad \boxed{\frac{1}{11}}$$

$$2. \quad \frac{50}{75} \quad \frac{\cancel{5} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{5} \cdot 3} \quad \boxed{\frac{2}{3}}$$

$$3. \quad \frac{21x^3y^2}{6xy^4} \quad \frac{\cancel{7} \cdot \cancel{3} x^3 y^2}{\cancel{2} \cdot \cancel{3} x y^4} \quad \boxed{\frac{7x^2}{2y^2}}$$

$$4. \quad \frac{36x^5y^3}{72x^4y^3} \quad \frac{\cancel{1} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} x^5 y^3}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} x^4 y^3} \quad \boxed{\frac{1x}{2}}$$

$$5. \quad \frac{x^2 - 3x}{x - 3} \quad \boxed{X}$$

$$\frac{x(x - \cancel{3})}{x \cancel{-3}}$$

$$6. \quad \frac{5x}{25x^2 - 30x} \quad \boxed{\frac{1}{5x - 6}}$$

$$\frac{\cancel{5}x}{5x(5x - 6)}$$

$$7. \quad \frac{x^2 + 2x}{x^2 + 3x + 2} \quad \boxed{\frac{x}{x+1}}$$

$$\frac{x(x + \cancel{2})}{(x + \cancel{2})(x + 1)}$$

$$8. \quad \frac{x^2 + 3x + xy + 3y}{x^2 + 5x + 6} \quad \boxed{\frac{x+y}{x+2}}$$

$$\frac{x^2 + 3x \quad xy + 3y}{x(x+3) \quad y(x+3)}$$

$$\frac{(x+y)(x+3)}{(x+2)(x+3)}$$

$$\frac{(x+y)\cancel{(x+3)}}{(x+2)\cancel{(x+3)}}$$

5.7: Simplify Rational Expressions Practice Problems Continue

Simplify:

9. $\frac{x+3}{x^2+x-6}$

$$\frac{\cancel{x+3}}{(\cancel{x+3})(x-2)} = \boxed{\frac{1}{x-2}}$$

10. $\frac{x^2-2x-15}{x^2-8x+15}$

$$\frac{(\cancel{x+3})(\cancel{x-5})}{(\cancel{x-5})(x-3)} = \boxed{\frac{x+3}{x-3}}$$

11. $\frac{x^2-16}{x^2-8x+16}$

$$\frac{(\cancel{x-4})(x+4)}{(x+4)(\cancel{x-4})} = \boxed{\frac{x+4}{x-4}}$$

12. $\frac{2x^2+11x+5}{2x^2-50}$

$$\frac{2x^2+11x+5}{2x^2-50} = \frac{2x^2+1x+10x+5}{2x^2-50}$$

$$\frac{x(2x+1) \quad 5(2x+1)}{(x+5)(2x+1)} = \frac{2(x^2-25)}{2(x-5)(x+5)}$$

$$\frac{(\cancel{x+5})(2x+1)}{2(x-5)(\cancel{x+5})} = \boxed{\frac{2x+1}{2(x-5)}}$$

13. $\frac{3x^2+13x+12}{x^2-4x-21}$

$$\frac{3x^2+13x+12}{x^2-4x-21} = \frac{3x^2+9x+4x+12}{(x+3)(x-7)}$$

$$\frac{3x(3x+4) \quad 4(x+3)}{(\cancel{x+3})(x-7)} = \boxed{\frac{3x+4}{x-7}}$$

14. $\frac{6x^2-7x-3}{8x^2-6x-9}$

$$\frac{6x^2-7x-3}{8x^2-6x-9} = \frac{6x^2+2x-9x-3}{8x^2+6x-12x-9}$$

$$\frac{2x(3x+1) \quad -3(3x+1)}{(2x-3)(3x+1)} = \frac{8x^2+6x-12x-9}{(2x-3)(4x+3)}$$

$$\frac{(\cancel{2x-3})(3x+1)}{(\cancel{2x-3})(4x+3)} = \boxed{\frac{3x+1}{4x+3}}$$



March 17th 2008

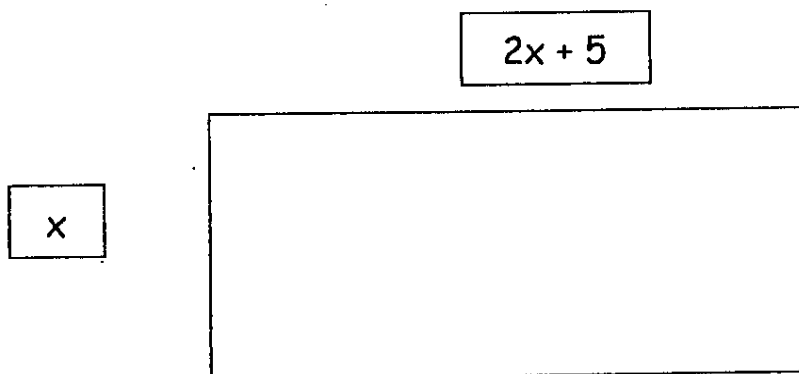
5.8: Factoring Applications

What are some examples where factoring is used in life?

Word Problem:

Find the area of a rectangle given that the length of the rectangle is 5 more than twice the width.

Solution:



To find the area of a rectangle use the formula for Area of a rectangle: Area = Length times Width or $A = L \cdot W$

In this case, the width is "x" and the length is "2x+5" so the area is:

$$A = L \cdot W$$

$$A = x(2x + 5)$$

$$A = 2x^2 + 5x$$

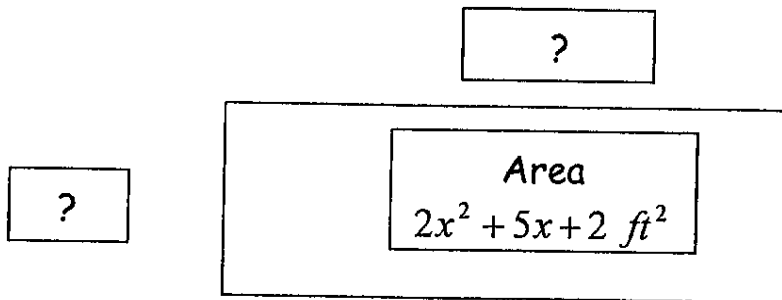
Factoring Application 1: Factoring can be used to find unknowns.

Example:

Find the length and width of a rectangle whose area is: $2x^2 + 5x + 2 \text{ ft}^2$

(In this case, the length is the longer side and the width is the shorter side.)

Once the width and length algebraic expressions are found, find the value of the width and length when $x = 5 \text{ ft}$. Then verify the area by substituting $x = 5 \text{ ft}$ into the area expression.



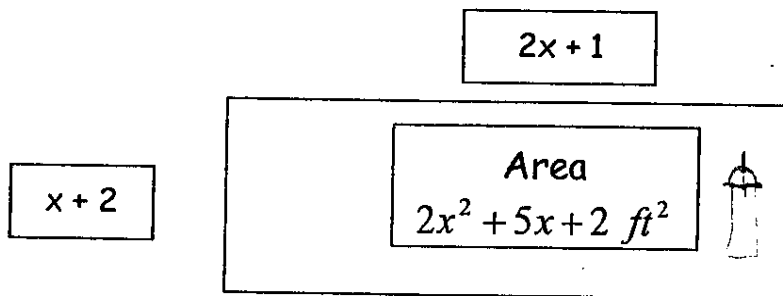
Solution:

To find the solution factor the area to find the length and width:

$$2x^2 + 5x + 2$$

$$(x + 2)(2x + 1)$$

The width is " $x + 2$ " feet and the length " $2x + 1$ " feet.

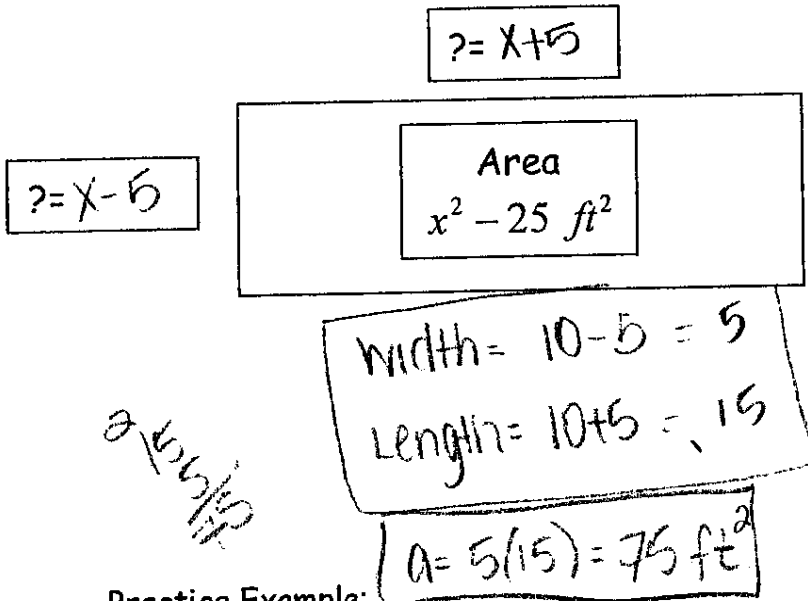


If $x = 5 \text{ ft}$, then:

| Width | Length | Area | Area |
|----------------|-----------------|---|------------------------------|
| $x + 2$ | $2x + 1$ | | $2x^2 + 5x + 2 \text{ ft}^2$ |
| $(5) + 2$ | $2(5) + 1$ | | $2(5)^2 + 5(5) + 2$ |
| 7 ft | 11 ft | $(7 \text{ ft})(11 \text{ ft}) = 77 \text{ ft}^2$ | $50 + 25 + 2$ |
| | | | 77 ft^2 |

Practice Example:

Find the length and width of a rectangle garden whose area is: $x^2 - 25 \text{ ft}^2$
 (In this case the length is the longer side and the width is the shorter side.)
 Once the width and length are found, find the value of the width and length when $x = 10 \text{ ft}$. Then verify the area by substituting $x = 10 \text{ ft}$ into the area.

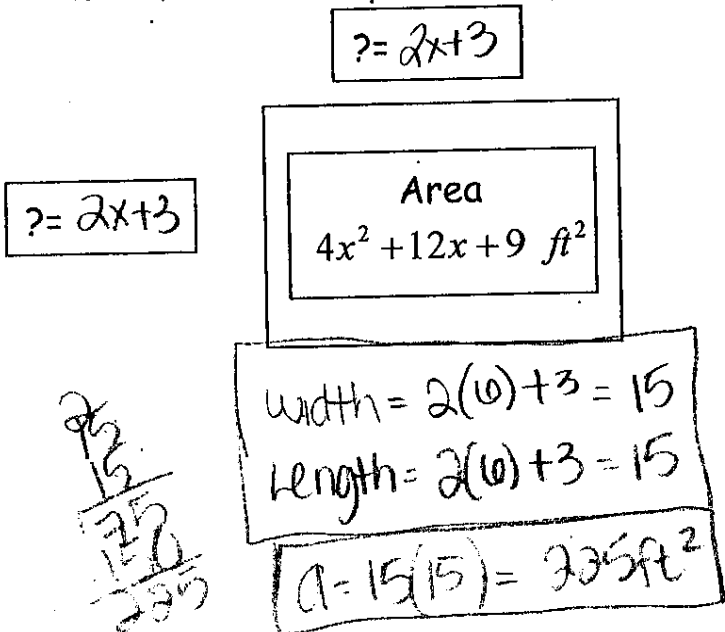


$x^2 - 25$ #1 #2 #3
 $(x + 5)(x - 5)$

$\sqrt{x^2 - 25}$
 $10^2 - 25$
 $100 - 25$
 75

Practice Example:

Find the length and width of a square play area whose area is: $4x^2 + 12x + 9 \text{ ft}^2$
 (In this case the length is the longer side and the width is the shorter side.)
 Once the width and length are found, find the value of the width and length when $x = 6 \text{ ft}$. Then verify the area by substituting $x = 6 \text{ ft}$ into the area.



$4x^2 + 12x + 9$
 $2x + 3$
 $(2x + 3)^2$

$\sqrt{4(6)^2 + 12(6) + 9}$
 $4(36) + 12(6) + 9$
 $144 + 72 + 9$
 225

36
 $\frac{4}{144}$
 $\frac{12}{72}$
 $\frac{1}{144}$
 $\frac{9}{225}$

Factoring Application 2: Solving real life equations



The equation of a person who dives off a 48 ft. cliff into a river with an initial velocity of 32 ft/sec is:

$h = -16t^2 + 32t + 48$, where "t" is time in seconds and "h" is height in feet.

When will the diver hit the water (when height = 0)?

Solution:

To find the solution:

1) Substitute 0 in for the height (h)

$$h = -16t^2 + 32t + 48$$

$$0 = -16t^2 + 32t + 48$$

$$-16t^2 + 32t + 48 = 0$$

2) Factor the trinomial by first factoring out the -16:

$$-16t^2 + 32t + 48 = 0$$

$$-16(t^2 - 2t - 3) = 0$$

3) Continue factoring by factoring the trinomial:

$$-16(t^2 - 2t - 3) = 0$$

$$-16(t-3)(t+1) = 0$$

4) Set each term with a variable equal to the right side of 0 and solve for t:

$$t-3=0 \quad t+1=0$$

$$+3 \quad +3 \quad -1 \quad -1$$

$$t=3 \quad t=-1$$

5) The solutions are 3 seconds and -1 seconds. Since -1 seconds is not realistic, we do not include that solution. The diver will hit the water in 3 seconds.



Practice Example:

The equation of a person who dives off a 64 ft. cliff into a river with an initial velocity of 0 ft/sec is:

$$h = -16t^2 + 64, \text{ where "t" is time in seconds and "h" is height in feet.}$$

When will the diver hit the water (when height = 0)?

$$0 = -16t^2 + 64$$

$$0 = -16(t^2 - 4) \begin{matrix} \#1 \\ \#2 \end{matrix} \begin{matrix} \#3 \\ \#3 \end{matrix}$$

$$0 = -16(t+2)(t-2)$$

2 sec

$$\begin{matrix} t+2=0 & t-2=0 \\ -2 & -2 \\ \hline t=-2 & t=2 \end{matrix}$$

$$\begin{array}{r} 1 \\ 16 \\ \hline 16 \\ 32 \\ \hline 16 \\ 48 \\ \hline 16 \\ 64 \end{array}$$

Practice Example:

The equation of the path of a ball thrown in the air at a speed of 96 ft/sec is:

$$h = -16t^2 + 96t, \text{ where "t" is time (seconds) and "h" is height in feet.}$$

When will the ball hit the ground (when height = 0)?

$$0 = -16t^2 + 96t$$

$$0 = -16t(t-6)$$

6 sec

$$\begin{matrix} -16t=0 & t-6=0 \\ -16 & -16 \\ \hline t=0 & t=6 \end{matrix}$$

$$\begin{array}{r} 1 \\ 16 \\ \hline 16 \\ 80 \\ \hline 16 \\ 96 \end{array}$$

What are some examples where factoring is used in life?

physics, - formulas, area, equations in life

5.8: Factoring Applications Practice Problems

Solve:

1. Find the length and width of a rectangular room whose area is: $16x^2 - 25 \text{ ft}^2$

Draw a picture of the situation.

(In this case the length is the longer side and the width is the shorter side.)

Once the width and length are found, find the value of the width and length when $x = 10 \text{ ft}$. Then verify the area by substituting $x = 10 \text{ ft}$ into the area.

$$x = 4x + 5$$

area
 $16x^2 - 25 \text{ ft}^2$ $x = 4x - 5$

$$16x^2 - 25$$

$$(4x - 5)(4x + 5)$$

$$\checkmark 16(10)^2 - 25$$

$$16(100) - 25$$

$$1600 - 25$$

$$1575$$

width = $4(10) - 5 = 35$

length = $4(10) + 5 = 45$

Area = $35(45) = 1575 \text{ ft}^2$

$$\begin{array}{r} 35 \\ 45 \\ \hline 1575 \end{array}$$

2. Find the length and width of a square room whose area is: $4x^2 + 20x + 25 \text{ ft}^2$

Draw a picture of the situation.

(In this case the length is the longer side and the width is the shorter side.)

Once the width and length are found, find the value of the width and length when $x = 4 \text{ ft}$. Then verify the area by substituting $x = 4 \text{ ft}$ into the area.

$$x = 2x + 5$$

Area
 $4x^2 + 20x + 25 \text{ ft}^2$ $x = 2x + 5$

$$4x^2 + 20x + 25$$

$$2x + 5$$

$$(2x + 5)^2$$

$$\checkmark 4(4)^2 + 20(4) + 25$$

$$4(16) + 20(4) + 25$$

$$64 + 80 + 25$$

$$169$$

$$\begin{array}{r} 2 \\ 16 \\ 4 \\ \hline 64 \\ 20 \\ 4 \\ \hline 80 \\ 64 \\ 80 \\ 25 \\ \hline 169 \end{array}$$

width = $2(4) + 5 = 13$

length = $2(4) + 5 = 13$

Area = $13(13) = 169 \text{ ft}^2$

$$\begin{array}{r} 13 \\ 13 \\ \hline 39 \\ 130 \\ \hline 169 \end{array}$$

5.8: Factoring Applications Practice Problems Continue

Solve:

3. The equation of a person who dives off a 96 ft. cliff into a river with an initial velocity of 16 ft/sec is:

$$h = -16t^2 + 16t + 96, \text{ where "t" is time in seconds and "h" is height in feet.}$$

When will the diver hit the water (when height = 0)?

$$0 = -16t^2 + 16t + 96$$

$$0 = -16(t^2 - t - 6)$$

$$0 = -16(t-3)(t+2)$$

$$\begin{array}{r} t-3=0 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\boxed{t=3}$$

$$\begin{array}{r} t+2=0 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\boxed{t=-2}$$

3 seconds

4. The equation of the path of a ball thrown in the air at a speed of 16 ft/sec is:

$$h = -16t^2 + 16t, \text{ where "t" is time (seconds) and "h" is height in feet.}$$

When will the ball hit the ground (when height = 0)?

$$0 = -16t^2 + 16t$$

$$0 = -16t(t-1)$$

$$\begin{array}{r} -16t=0 \\ -16 \quad -16 \\ \hline \end{array}$$

$$\boxed{t=0}$$

$$\begin{array}{r} t-1=0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$\boxed{t=1}$$

1 second

March 15, 2008

MAT0024C Beginning Algebra

Name: Nicole Trevisan

Test 3 (Chapter 5) REVIEW

Questions from Chapter 5

Factor:

1. $10x^6 - 6x^8$

$2x^6(5 - 3x^2)$

$10x^6 - 6x^8$

2. $24v^4w^4 + 4vw^2$

$4vw^2(6v^3w^2 + 1)$

$24v^4w^4 + 4vw^2$

3. $45x^6y^{55} + 10x^{18}y^{15} + 55x^{24}y^{35}$

$5x^6y^{15}(9y^{40} + 2x^{12} + 11x^{18}y^{20})$

$45x^6y^{55} + 10x^{18}y^{15} + 55x^{24}y^{35}$

4. $2x^2 + 8x + 3x + 12$

$2x^2 + 8x \quad 3x + 12$
 $2x(x + 4) \quad 3(x + 4)$

$(2x + 3)(x + 4)$

$2x^2 + 8x + 3x + 12$

5. $7x^3 - 21x^2 + x - 3$

$7x^3 - 21x^2 \quad x - 3$
 $7x^2(x - 3) \quad 1(x - 3)$

$(7x^2 + 1)(x - 3)$

$7x^3 - 21x^2 + x - 3$

6. $15y^2 - 10yz + 6y - 4z$

$15y^2 - 10yz \quad 6y - 4z$
 $5y(3y - 2z) \quad 2(3y - 2z)$

$(5y + 2)(3y - 2z)$

$15y^2 - 10yz + 6y - 4z$

7. $x^2 + 11x + 30$ ⁺³⁰ ~~-11~~ ~~10~~ ^{5,6}

$(x-5)(x-6)$

✓ $x^2 - 6x - 5x + 30$
 $x^2 - 11x + 30$

8. $x^2 - 8x + 16$ ⁺¹⁶ ⁻⁸ ^{8,2} ^{4,4}

$(x-4)(x-4)$

① $x^2 - 4x - 4x + 16$
 $x^2 - 8x + 16$

9. $x^2 - x - 6$ ⁻⁶ ⁻¹ ~~1~~ ^{2,3}

$(x+2)(x-3)$

✓ $x^2 - 3x + 2x - 6$
 $x^2 - x - 6$

10. $4c^2 + 12c + 9$ ⁺³⁰ ⁺¹² ^{12,3} ^{6,6}

$4c^2 + 6c + 6c + 9$

$4c^2 + 6c$ $6c + 9$
 $2c(2c+3)$ $3(2c+3)$
 $(2c+3)(2c+3)$

① $4c^2 + 6c + 6c + 9$ $4c^2 + 12c + 9$

11. $9n^2 - 9n - 10$ ⁻⁹⁰ ⁻⁹ ~~10~~ ^{5,3} ^{15,2}

$9n^2 - 6n + 15n - 10$

$9n^2 - 6n$ $15n - 10$
 $3n(3n-2)$ $5(3n-2)$

$(3n+5)(3n-2)$

✓ $9n^2 - 6n + 15n - 10$ $9n^2 - 9n - 10$

13. $16z^2 - 25$ ^{#1} ^{#2} ^{#3}

$(4z+5)(4z-5)$

① $16z^2 - 20z + 20z - 25$
 $16z^2 - 25$

12. $12x^2 + 19x + 5$ ⁶⁰ ¹² ⁵ ^{30,2} ^{5,15}

$12x^2 + 4x + 15x + 5$

$12x^2 + 4x$ $15x + 5$
 $4x(3x+1)$ $5(3x+1)$
 $(4x+5)(3x+1)$

① $12x^2 + 4x + 15x + 5$ $12x^2 + 19x + 5$

14. $2b^2 - 2c^2$

$2(b^2 - c^2)$ ^{#1} ^{#2} ^{#3}

$2(b+c)(b-c)$

① $(2b+2c)(b-c)$
 $2b^2 - 2bc + 2bc - 2c^2$
 $2b^2 - 2c^2$

Solve:

15. $x^2 - 8x = 0$

$x(x - 8) = 0$

$x = 0 \quad x - 8 = 0$
 $+8 \quad +8$

$x = 8$

$x = 0 \quad x = 8$

16. $x^2 - 4x - 5 = 0$

$(x - 1)(x - 5) = 0$

$x - 1 = 0 \quad x - 5 = 0$
 $+1 \quad +5$

$x = 1 \quad x = 5$

$x = -1 \quad x = 5$

Solve:

17. $3x^2 + 2x - 8 = 0$

$3x^2 + 6x - 4x - 8$

$3x(x + 2) - 4(x + 2)$

$3x(x + 2) - 4(x + 2)$

$(3x - 4)(x + 2) = 0$

$x = -2$
 $x = \frac{4}{3}$

$3x - 4 = 0 \quad x + 2 = 0$
 $+4 \quad +4 \quad -2 \quad -2$

$x = -2$

$\frac{3x}{3} = \frac{4}{3}$
 $x = \frac{4}{3}$

19. Simplify: $\frac{x^2 - x - 56}{x^2 - 49}$

$(x + 7)(x - 8)$
 $(x - 7)(x + 7)$

$\frac{(x + 7)(x - 8)}{(x - 7)(x + 7)}$

$\frac{x - 8}{x - 7}$

18. $4x^2 + 3x - 1 = 0$

$4x^2 + 4x - 1x - 1$

$4x(x + 1) - 1(x + 1)$

$4x(x + 1) - 1(x + 1)$

$(4x - 1)(x + 1) = 0$

$x = \frac{1}{4}$
 $x = -1$

$4x - 1 = 0 \quad x + 1 = 0$
 $+1 \quad +1 \quad -1 \quad -1$

$\frac{4x}{4} = \frac{1}{4}$
 $x = \frac{1}{4}$

20. Simplify: $\frac{x^2 - 4x - 12}{2x^2 - 15x + 18}$

$(x + 2)(x - 6)$
 $(2x - 3)(x - 6)$

$\frac{(x + 2)(x - 6)}{(2x - 3)(x - 6)}$

$\frac{x + 2}{2x - 3}$

21. Two machines can complete 8 tasks every 3 days. Let t represent the number of tasks these machines can complete in a 30-day month. Write a proportion to show this example.

$$\frac{8}{3} = \frac{t}{30}$$

22. Simplify: $7x + 8(x - 3)$

$$\begin{array}{r} 7x + 8x - 24 \\ \hline 15x - 24 \end{array}$$

23. Convert to scientific notation: 0.000023

$$\underbrace{000023}_5$$

$$2.3 \times 10^{-5}$$

24. Solve: $2x - 10 < -8$

$$\begin{array}{r} 2x - 10 < -8 \\ +10 \quad +10 \end{array}$$

$$\frac{2x}{2} < \frac{2}{2}$$

$$x = 1$$

25. Simplify: $(a^2b^4)^3(a^3b^4)$

$$(a^6b^{12})(a^3b^4)$$

$$a^9b^{16}$$

26. Simplify: $|8 + (-14)| + 9$

$$|-6| + 9$$

$$6 + 9$$

$$15$$

27. Solve for t: $x = -8z + 7t$
 $+ 8z + 8z$

$$\frac{8z + x}{7} = \frac{7t}{7}$$

$$t = \frac{8}{7}z + \frac{1}{7}x$$

28. Find the y-intercept for: $-5x + 8y = -8$

$$x = 0$$

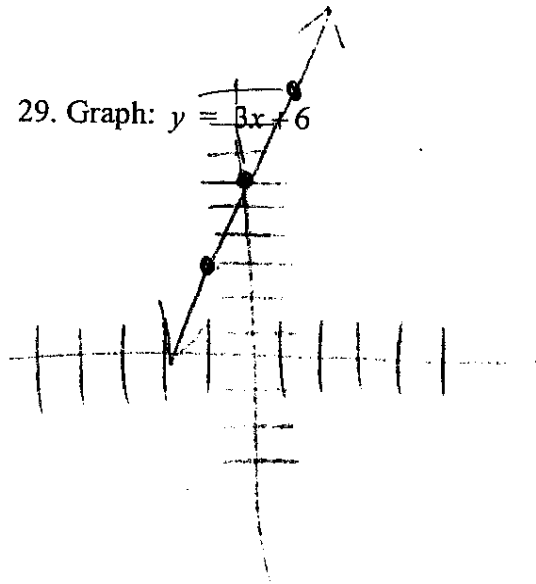
$$-5(0) + 8y = -8$$

$$\frac{8y}{8} = \frac{-8}{8}$$

$$y = -1$$

$$y \text{ intercept} = -1$$

29. Graph: $y = 3x + 6$



30. The length of a rectangular pool is 6 less than twice the width. The perimeter of the pool is 78 feet. Find the length and width of the pool. Label each distance correctly.

$$L = 2W - 6 \quad \begin{matrix} 2(15) - 6 \\ 30 - 6 \end{matrix}$$

$$W = 15$$

$$L = 24$$

$$78 = P$$

~~$$2L + 2W = 78$$~~

$$6W - 12 = 78$$

$$+ 12 \quad + 12$$

$$6W = 90$$

$$\frac{6W}{6} = \frac{90}{6}$$

$$W = 15$$

$$\text{Length} = 24 \text{ ft}$$

$$\text{width} = 15 \text{ ft}$$